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University of Niš.

PREFACE

Dear Colleagues,

The University of Niš was founded more than 30 years ago and nowadays consists of ten faculties: the Faculty of Civil Engineering, the Faculty of Electronics, the Faculty of Mechanical Engineering, the Faculty of Technology, the Faculty of Economics, the Faculty of Law, the Faculty of Medicine, the Faculty of Philosophy, the Faculty of Occupational Safety and the Teacher-Training Faculty.

In 1985, the administrative bodies of the University initiated the publication of the scientific journal FACTA UNIVERSITATIS. According to the criteria adopted, the scientific papers accepted for publishing in the Journal must be reviewed and written in English, French, German or Russian language. The Journal is anticipated to appear periodically through several series covering major disciplines studied at the University. Accordingly, the Journal is conceptualized as a compilation of 14 separate series:

Series I	Economics and Organization
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The University Library "Nikola Tesla" maintains the exchange of the published series for more than 200 similar scientific periodical titles from more than 25 countries from all over the world: Austria, Belgium, Bulgaria, Denmark, Finland, France, Greece, Sweden, Brazil, U.S.A., Canada, Japan, China, Taiwan, Malaysia, New Zealand etc.

The duties of the editor-in-chief were formerly performed by Prof. dr. Radosav Djordjević (1985-1987) and Prof. dr. Gradimir Milovanović (1987-1989).

The Series "Mechanics, Automatic Control and Robotics" is aimed at publishing scholarly papers in the field of theoretical and applied mechanics and general engineering knowledge fundamental for mechanical engineering, electrotechnics and civil engineering in the area of dynamic systems, static and dynamics of constructions and machines, kinematics and dynamics of machines and mechanisms, automatic control and kinematics and dynamics of robots and manipulators The papers intended for publication must be reviewed and should present original contributions in the field they deal with. The journal is also open for review papers, scholarly papers presenting original interpretations of

already established postulates in the afore-mentioned fields, as well as the papers concerned with teaching and research methodology, history and philosophy of natural and Engineering science with appropriate applications.

Some of the papers published in our Journal were presented on the Yugoslav Scientific Conference on Mechanics and other scientific seminars held at the University of Niš, University of Belgrade and at the Mathematical Institute of the Serbian Academy of Arts and Sciences in Belgrade.

Editor will gladly receive all contributions of the scholars showing interest for having their research results presented in our Journal, provided that they are written in accordance with the enclosed instruction for authors. We are also permanently open for your contributions presenting current information on scientific meetings, as well as for reviews of monographs, studies and other scholarly publications in the cited fields.

The authors should preferably submit their manuscripts in English. Thank you in advance for future contacts and collaboration.

The papers published so far in this Series have been reported and reviewed in the following reviews: "Реферативний журнал - Механика" - Moscow, "Mathematical Reviews", "Applied Mechanics Reviews", "Zentralblat", and other publications, while some of the papers may also be found in the data-bases of certain information centers

Katica (Stevanović) Hedrih, Editor-in Chief of the Journal FACTA UNIVERSITATIS



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BOUNDARY LAYER OF IONIZED GAS IN THE CASE OF CHANGEABLE ELECTROCONDUCTIVITY

UDC 593.951: 537.56

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Abstract. This paper studies ionized gas flow in the case of a concrete form of the law of its electroconductivity change. The corresponding boundary layer equations are first by means of suitable transformations brought to a universal form. Then the equations are numerically solved in three-parametric approximation. Physical values and some boundary layer characteristics are graphically presented.

1. INTRODUCTORY STUDIES, STARTING EQUATIONS

This paper studies ionized gas flow in the boundary layer on the body of arbitrary shape. The study represents the sequence of our previous analysis of this extremely complex case of the fluid flow. Ionized gas, that is plasma, is essentially different from the three known aggregate states; therefore it is usually called the fourth aggregate state.

One of the main features of ionized gas is its electroconductivity. As a matter of fact, due to ionization, and under the influence of the outer magnetic field, an electric stream appears in the gas. The electric stream causes the appearance of Lorentz's force and Joule's heat. Because of these two effects new members appear in the corresponding boundary layer equations, which is not the case in the equations of homogeneous gas which is not ionized. Therefore, in the case of ionized gas flow, equations of laminar, steady and plane boundary layer [1], [2], in the conditions of so-called equilibrium ionization, have the following form:

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$$\frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) = 0,$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{dp}{dx} + \frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial y} \right) - \sigma B_m^2 u,$$

$$\rho u \frac{\partial h}{\partial x} + \rho v \frac{\partial h}{\partial y} = u \frac{dp}{dx} + \mu \left(\frac{\partial u}{\partial y} \right)^2 + \frac{\partial}{\partial y} \left(\frac{\mu}{\Pr} \frac{\partial h}{\partial y} \right) + \sigma B_m^2 u^2;$$

$$u = v = 0 \quad h = h_w \quad \text{for} \quad y = 0,$$

$$u \to u_e(x), \quad h \to h_e(x) \quad \text{for} \quad y \to \infty,$$

$$(u = u_0(y), \quad h = h_0(y) \quad \text{for} \quad x = x_0)$$

In these equations as well as in the corresponding boundary conditions, the symbols common in the boundary layer theory are used for certain physical values. So, $\sigma B_m^2 u^2$ represents Joule's heat. It is considered, as usually [1], that the outer magnetic field is perpendicular to the contour of the body, and that because of a relatively small thickness of the boundary layer the equation of this field is $B_m = B_m(x)$.

In our earlier studies it was presumed that the electroconductivity of ionized gas could also be presented in a form of a function only of longitudinal coordinate x, i.e. $\sigma = \sigma(x)$. However, some of the characteristics and parametric solutions of the ionized gas boundary layer equations, obtained according to this form of electroconductivity law [3], show that, among other things, it is necessary to have a concrete form of this law. Therefore, in this paper and in this phase of our studies, it is considered that the law of electroconductivity change is determined by the expression [4, 6]:

$$\sigma = \sigma_0 \left(1 - \frac{u}{u_e} \right), \qquad \sigma_0 = const. \tag{2}$$

According to the form of the law (2), it is concluded that electroconductivity disappears at the outer boundary of the boundary layer, i.e. $\sigma = \sigma_e = 0$ at this boundary.

If, by the known procedure, we exclude the pressure from the equation system (1) then the system can take the following form:

$$\frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) = 0,$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \rho_e u_e \frac{du_e}{dx} + \frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial y} \right) - \sigma B_m^2 u,$$

$$\rho u \frac{\partial h}{\partial x} + \rho v \frac{\partial h}{\partial y} = -u \rho_e u_e \frac{du_e}{dx} + \mu \left(\frac{\partial u}{\partial y} \right)^2 + \frac{\partial}{\partial y} \left(\frac{\mu}{\Pr} \frac{\partial h}{\partial y} \right) + \sigma B_m^2 u^2;$$

$$(1')$$

with the unchanged boundary conditions. Here, the index "e" presents physical values at the edge of the boundary layer.

2. Introduction of New Variables

Modern parametric methods of solution of boundary layer equations are based, as it is known, on the application of an impulse equation. In order for an impulse equation to have the simplest form we introduce, as with similar flow problems [2], new variables s, z and the stream function ψ in the form of:

$$s(x) = \frac{1}{\rho_0 \mu_0} \int_0^x \rho_w \mu_w \, dx, \qquad z(x, y) = \frac{1}{\rho_0} \int_0^y \rho \, dy$$

$$u = \frac{\partial \psi}{\partial z}, \qquad \widetilde{v} = \frac{\rho_0 \mu_0}{\rho_w \mu_w} \left(u \frac{\partial z}{\partial x} + v \frac{\rho}{\rho_0} \right) = -\frac{\partial \psi}{\partial s}$$
(3)

where the index "w" represents conditions at the wall of the body rounded with fluid, while ρ_0 , $\mu_0 = \rho_0 v_0$ represent the known constant values of density and dynamic viscosity of ionized gas.

By means of newly introduced transformations (3) the starting equation system (1') comes down to:

$$\frac{\partial \psi}{\partial z} \frac{\partial^{2} \psi}{\partial s \partial z} - \frac{\partial \psi}{\partial s} \frac{\partial^{2} \psi}{\partial z^{2}} = \frac{\rho_{e}}{\rho} u_{e} \frac{du_{e}}{ds} + v_{0} \frac{\partial}{\partial z} \left(Q \frac{\partial^{2} \psi}{\partial z^{2}} \right) - \frac{\rho_{0} \mu_{0}}{\rho_{w} \mu_{w}} \frac{\sigma B_{m}^{2}}{\rho} \frac{\partial \psi}{\partial z},$$

$$\frac{\partial \psi}{\partial z} \frac{\partial h}{\partial s} - \frac{\partial \psi}{\partial s} \frac{\partial h}{\partial z} = -\frac{\rho_{e}}{\rho} u_{e} \frac{du_{e}}{ds} \frac{\partial \psi}{\partial z} + v_{0} Q \left(\frac{\partial^{2} \psi}{\partial z^{2}} \right)^{2} + v_{0} \frac{\partial}{\partial z} \left(\frac{Q}{\Pr} \frac{\partial h}{\partial z} \right) + \frac{\rho_{0} \mu_{0}}{\rho_{w} \mu_{w}} \frac{\sigma B_{m}^{2}}{\rho} \left(\frac{\partial \psi}{\partial z} \right)^{2};$$

$$\psi = \frac{\partial \psi}{\partial z} = 0, \qquad h = h_{w} \qquad for \qquad z = 0,$$

$$\frac{\partial \psi}{\partial z} \rightarrow u_{e}(s) \qquad h \rightarrow h_{e}(s) \qquad for \qquad z \rightarrow \infty,$$

$$\left(\frac{\partial \psi}{\partial z} = u_{0}(z) \qquad h = h_{0}(z) \qquad for \qquad s = s_{0} \right).$$

$$(4)$$

In the equation system (4) the non-dimensional function Q and Prandtl's number Pr are determined by the following expressions:

$$Q = \frac{\rho \mu}{\rho_w \mu_w} , \qquad Pr = \frac{\mu c_p}{\lambda} . \qquad (5)$$

By means of the variables (3) and by the known procedure it is relatively easy to obtain the impulse equation based on the two equations of the system (1'). This impulse equation can be written in its three form as:

$$\frac{dZ^{**}}{ds} = \frac{F_m}{u_e} , \qquad \frac{df}{ds} = \frac{u'_e}{u_e} F_m + \frac{u''_e}{u'_e} f , \qquad \frac{\Delta^{**'}}{\Delta^{**}} = \frac{u'_e}{u_e} \frac{F_m}{2f} ; \qquad (6)$$

where the apostrophe represents the derivative per the longitudinal variable s.

While obtaining the impulse equation we take into consideration the parameter f, the magnetic parameter g, the conditional thickness Δ_1^{**} as well as the other common values

and characteristics of the boundary layer, which are:

$$Z^{**} = \frac{\Delta^{**}^{2}}{v_{0}}, \qquad f(s) = f_{1} = u'_{e} Z^{**}, \qquad g(s) = g_{1} = N_{\sigma} Z^{**},$$

$$F_{m} = 2\left[\zeta - (2+H)f\right] + 2gH_{1}, \qquad \zeta = \left[\frac{\partial (u/u_{e})}{\partial (z/\Delta^{**})}\right]_{z=0}, \qquad H = \frac{\Delta^{*}}{\Delta^{**}},$$

$$N_{\sigma}(s) = \frac{\rho_{0} \mu_{0}}{\rho_{w} \mu_{w}} N, \qquad N = \frac{\sigma_{0} B_{m}^{2}}{\rho_{e}}; \qquad \Delta^{*}(s) = \int_{0}^{\infty} \left(\frac{\rho_{e}}{\rho} - \frac{u}{u_{e}}\right) dz, \qquad H_{1} = \frac{\Delta_{1}^{**}}{\Delta^{**}},$$

$$\Delta^{**}(s) = \int_{0}^{\infty} \frac{u}{u_{e}} \left(1 - \frac{u}{u_{e}}\right) dz; \qquad \Delta_{1}^{**}(s) = \int_{0}^{\infty} \frac{u}{u_{e}} \left(1 - \frac{u}{u_{e}}\right) \frac{\rho_{e}}{\rho} dz.$$

$$(7)$$

The tangential stress at the wall of the body rounded with fluid, in this case of flow, is determined by the non-dimensional function of friction ζ as:

$$\tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{v=0} = \frac{\rho_w \mu_w}{\rho_0} \frac{u_e}{\Delta^{**}} \zeta. \tag{7'}$$

Using the ideas from [2] we apply another transformation of variables to the equation system (4):

$$s = s, \eta(s, z) = \frac{B(s)}{\Delta^{**}(s)} z, \psi(s, z) = \frac{u_e(s) \Delta^{**}(s)}{B(s)} \Phi(s, \eta),$$

$$h(s, z) = h_l \cdot \overline{h}(s, \eta); h_e + \frac{u_e^2}{2} = h_l = const.;$$
(8)

where $\Phi(s,z)$ and $\overline{h}(s,\eta)$ stand for the conditional stream function and the non-dimensional enthalpy.

The previously introduced relations and characteristics (7) can be, by means of the newly introduced transformations (8), written as:

$$\zeta = B \left(\frac{\partial^{2} \Phi}{\partial \eta^{2}} \right)_{\eta=0}; \qquad H = \frac{A}{B}, \qquad H_{1} = \frac{A_{1}}{B}, \qquad B = \int_{0}^{\infty} \frac{\partial \Phi}{\partial \eta} \left(1 - \frac{\partial \Phi}{\partial \eta} \right) d\eta,
A = \int_{0}^{\infty} \left(\frac{\rho_{e}}{\rho} - \frac{\partial \Phi}{\partial \eta} \right) d\eta, \qquad A_{1} = \int_{0}^{\infty} \frac{\partial \Phi}{\partial \eta} \left(1 - \frac{\partial \Phi}{\partial \eta} \right) \frac{\rho_{e}}{\rho} d\eta;$$
(9)

where it is presumed that the values A, A_1 and B are continuous functions of the coordinate s.

The equation system (4), after a complex derivation and by means of the transformations (8), comes down to:

$$\frac{\partial}{\partial \eta} \left(\mathcal{Q} \frac{\partial^{2} \Phi}{\partial \eta^{2}} \right) + \frac{aB^{2} + (2 - b) f}{2B^{2}} \Phi \frac{\partial^{2} \Phi}{\partial \eta^{2}} + \frac{f}{B^{2}} \left[\frac{\rho_{e}}{\rho} - \left(\frac{\partial \Phi}{\partial \eta} \right)^{2} \right] - \frac{g}{B^{2}} \frac{\rho_{e}}{\rho} \left(1 - \frac{\partial \Phi}{\partial \eta} \right) \frac{\partial \Phi}{\partial \eta} =$$

$$= \frac{u_{e} Z^{**}}{B^{2}} \left(\frac{\partial \Phi}{\partial \eta} \frac{\partial^{2} \Phi}{\partial s \partial \eta} - \frac{\partial \Phi}{\partial s} \frac{\partial^{2} \Phi}{\partial \eta^{2}} \right),$$

$$\frac{\partial}{\partial \eta} \left(\frac{\mathcal{Q}}{\Pr} \frac{\partial \overline{h}}{\partial \eta} \right) + \frac{aB^{2} + (2 - b) f}{2B^{2}} \Phi \frac{\partial \overline{h}}{\partial \eta} - \frac{2\kappa f}{B^{2}} \frac{\rho_{e}}{\rho} \frac{\partial \Phi}{\partial \eta} + 2\kappa \mathcal{Q} \left(\frac{\partial^{2} \Phi}{\partial \eta^{2}} \right)^{2} + \frac{2\kappa g}{B^{2}} \frac{\rho_{e}}{\rho} \left(1 - \frac{\partial \Phi}{\partial \eta} \right) \left(\frac{\partial \Phi}{\partial \eta} \right)^{2} =$$

$$= \frac{u_{e} Z^{**}}{B^{2}} \left(\frac{\partial \Phi}{\partial \eta} \frac{\partial \overline{h}}{\partial s} - \frac{\partial \Phi}{\partial s} \frac{\partial \overline{h}}{\partial \eta} \right);$$

$$\Phi = \frac{\partial \Phi}{\partial \eta} = 0, \qquad \overline{h} = \overline{h}_{w} \qquad for \qquad \eta = 0,$$

$$\frac{\partial \Phi}{\partial \eta} \to 1, \qquad \overline{h} = \overline{h}_{e} = 1 - \kappa \qquad for \qquad \eta \to \infty,$$

$$(\Phi = \Phi_{0}(\eta), \qquad \overline{h} = \overline{h}_{0}(\eta) \qquad for \qquad s = s_{0})$$

It is pointed out that the obtained equation system (10) is different from the corresponding equations [3], in the case when $\sigma = \sigma(x)$, only in sign and in the form of the last member on the left handside of the sign of equality of the both equations (underlined members).

Further on, the local parameter of compressibility $\kappa = f_0$ and constants a, b exist in the equation systems (10) and in the corresponding boundary conditions. The parameter of compressibility satisfies the simple differential equation:

$$\kappa = \frac{u_e^2}{2h_1}, \qquad u_e Z^{**} \frac{d\kappa}{ds} = 2\kappa f \equiv \theta_0.$$
 (11)

3. GENERALIZATION OF BOUNDARY LAYER EQUATIONS OF THE CONSIDERED PROBLEM AND THEIR SOLUTIONS

It is also noticed that, besides the parameters and their distributions: κ , f, g, Pr, Q, ρ_e/ρ , the outer velocity $u_e(s)$ clearly exists in the obtained equation system (10). Therefore the solution of the system depends on each concrete form of distribution of this velocity. In order to avoid the noticed disadvantage instead of the transformations (8) we apply the similarity transformations in the following form:

$$\psi(s,z) = \frac{u_e \,\Delta^{**}}{B} \,\Phi\left(\eta, \kappa, f, g\right); \qquad h = h_l \cdot \overline{h} \,(\eta, \kappa, f, g). \tag{12}$$

With the new transformations (12), the function Φ and the non-dimensional enthalpy \overline{h} are not directly dependent on the longitudinal variable s but indirectly by means of the parameters κ , f and g.

By means of the relations (12) the basic equation system (4) is transformed into this form:

$$\frac{\partial}{\partial \eta} \left(\mathcal{Q} \frac{\partial^{2} \Phi}{\partial \eta^{2}} \right) + \frac{aB^{2} + (2 - b)f}{2B^{2}} \Phi \frac{\partial^{2} \Phi}{\partial \eta^{2}} + \frac{f}{B^{2}} \left[\frac{\rho_{e}}{\rho} - \left(\frac{\partial \Phi}{\partial \eta} \right)^{2} \right] - \frac{g}{B^{2}} \frac{\rho_{e}}{\rho} \left(1 - \frac{\partial \Phi}{\partial \eta} \right) \frac{\partial \Phi}{\partial \eta} =$$

$$= \frac{u_{e}Z^{**}}{B^{2}} \left[\left(\frac{\partial \Phi}{\partial \eta} \frac{\partial^{2} \Phi}{\partial \eta \partial \kappa} - \frac{\partial \Phi}{\partial \kappa} \frac{\partial^{2} \Phi}{\partial \eta^{2}} \right) \kappa' + \left(\frac{\partial \Phi}{\partial \eta} \frac{\partial^{2} \Phi}{\partial \eta \partial f} - \frac{\partial \Phi}{\partial f} \frac{\partial^{2} \Phi}{\partial \eta^{2}} \right) f' + \left(\frac{\partial \Phi}{\partial \eta} \frac{\partial^{2} \Phi}{\partial \eta \partial g} - \frac{\partial \Phi}{\partial g} \frac{\partial^{2} \Phi}{\partial \eta^{2}} \right) g' \right], \quad (13)$$

$$\frac{\partial}{\partial \eta} \left(\frac{Q}{Pr} \frac{\partial \overline{h}}{\partial \eta} \right) + \frac{aB^{2} + (2 - b)f}{2B^{2}} \Phi \frac{\partial \overline{h}}{\partial \eta} - \frac{2\kappa f}{B^{2}} \frac{\rho_{e}}{\rho} \frac{\partial \Phi}{\partial \eta} + 2\kappa Q \left(\frac{\partial^{2} \Phi}{\partial \eta^{2}} \right)^{2} + \frac{2\kappa g}{B^{2}} \frac{\rho_{e}}{\rho} \left(1 - \frac{\partial \Phi}{\partial \eta} \right) \left(\frac{\partial \Phi}{\partial \eta} \right)^{2} =$$

$$= \frac{u_{e}Z^{**}}{B^{2}} \left[\left(\frac{\partial \Phi}{\partial \eta} \frac{\partial \overline{h}}{\partial \kappa} - \frac{\partial \Phi}{\partial \kappa} \frac{\partial \overline{h}}{\partial \eta} \right) \kappa' + \left(\frac{\partial \Phi}{\partial \eta} \frac{\partial \overline{h}}{\partial f} - \frac{\partial \Phi}{\partial f} \frac{\partial \overline{h}}{\partial \eta} \right) f' + \left(\frac{\partial \Phi}{\partial \eta} \frac{\partial \overline{h}}{\partial g} - \frac{\partial \Phi}{\partial g} \frac{\partial \overline{h}}{\partial \eta} \right) g' \right].$$

As it is seen not even these equations can satisfy the conditions of generalized similarity because the members $u_eZ^{**}f'$ and $u_eZ^{**}g'$ contain $u_e(s)$ on their right hand sides and cannot be expressed by means of κ , f, g and F_m . Therefore in further researches, new parameters are introduced. As a matter of fact, the parameters f and g, used so far, were considered to be the first in order $f = f_1$ and $g = g_1$, while the introduced parameters f_2 and g_2 are the second order and they contain higher derivatives of the outer velocity u_e and of the function N_g . So, we have applied similarity transformations in this form:

$$\psi(s,z) = \frac{u_e \, \Delta^{**}}{B} \, \Phi(\eta, \, \kappa, \, f_1, \, f_2, \, g_1, \, g_2), \qquad h(s,z) = h_1 \cdot \overline{h} \, (\eta, \, \kappa, \, f_1, \, f_2, \, g_1, \, g_2).$$
 (12')

It has been shown that, in this case as well, the obtained equations contain new factors among which there are the outer velocity and its derivatives. Further detailed analysis has determined that also in this case of ionized gas flow it is necessary to introduce two sets of parameters in the following form:

$$f_k = u_e^{k-1} u_e^{(k)} Z^{**k}$$
, $f_0 = \kappa = \frac{u_e^2}{2h_1}$, $g_k = u_e^{k-1} N_\sigma^{(k-1)} Z^{**k}$, $(k = 1, 2, 3, ...)$ (14)

These parameters satisfy the next known recurring simple differential equations:

$$u_{e} Z^{**} f'_{k} = [(k-1) f_{1} + k F_{m}] f_{k} + f_{k+1} \equiv \theta_{k} ,$$

$$u_{e} Z^{**} g'_{k} = [(k-1) f_{1} + k F_{m}] g_{k} + g_{k+1} \equiv G_{k} .$$

$$(k = 1, 2, 3, ...)$$
(15)

If at the very beginning of the similarity transformations (12) we introduce the two sets of parameters (f_k) , (g_k) ; i.e. if we apply the transformations to the system (4):

$$\psi(s,z) = \frac{u_e \, \Delta^{**}}{R} \, \Phi[\, \eta, \, \kappa, \, (f_k), \, (g_k)], \qquad h = h_1 \cdot \, \overline{h} \, [\, \eta, \, \kappa, \, (f_k), \, (g_k)]$$
 (16)

then the system takes this form:

$$\frac{\partial}{\partial \eta} \left(\mathcal{Q} \frac{\partial^{2} \Phi}{\partial \eta^{2}} \right) + \frac{aB^{2} + (2 - b)f_{1}}{2B^{2}} \Phi \frac{\partial^{2} \Phi}{\partial \eta^{2}} + \frac{f_{1}}{B^{2}} \left[\frac{\rho_{e}}{\rho} - \left(\frac{\partial \Phi}{\partial \eta} \right)^{2} \right] - \frac{g_{1}}{B^{2}} \frac{\rho_{e}}{\rho} \left(1 - \frac{\partial \Phi}{\partial \eta} \right) \frac{\partial \Phi}{\partial \eta} =$$

$$= \frac{1}{B^{2}} \left[\sum_{k=0}^{\infty} \theta_{k} \left(\frac{\partial \Phi}{\partial \eta} \frac{\partial^{2} \Phi}{\partial \eta \partial f_{k}} - \frac{\partial \Phi}{\partial f_{k}} \frac{\partial^{2} \Phi}{\partial \eta^{2}} \right) + \sum_{k=1}^{\infty} G_{k} \left(\frac{\partial \Phi}{\partial \eta} \frac{\partial^{2} \Phi}{\partial \eta \partial g_{k}} - \frac{\partial \Phi}{\partial g_{k}} \frac{\partial^{2} \Phi}{\partial \eta^{2}} \right) \right],$$

$$\frac{\partial}{\partial \eta} \left(\frac{Q}{\Pr} \frac{\partial \overline{h}}{\partial \eta} \right) + \frac{aB^{2} + (2 - b) f_{1}}{2B^{2}} \Phi \frac{\partial \overline{h}}{\partial \eta} - \frac{2\kappa f_{1}}{B^{2}} \frac{\rho_{e}}{\rho} \frac{\partial \Phi}{\partial \eta} + 2\kappa Q \left(\frac{\partial^{2} \Phi}{\partial \eta^{2}} \right)^{2} + \frac{2\kappa g_{1}}{B^{2}} \frac{\rho_{e}}{\rho} \left(1 - \frac{\partial \Phi}{\partial \eta} \right) \left(\frac{\partial \Phi}{\partial \eta} \right)^{2} =$$

$$= \frac{1}{B^{2}} \left[\sum_{k=0}^{\infty} \theta_{k} \left(\frac{\partial \Phi}{\partial \eta} \frac{\partial \overline{h}}{\partial f_{k}} - \frac{\partial \Phi}{\partial f_{k}} \frac{\partial \overline{h}}{\partial \eta} \right) + \sum_{k=1}^{\infty} G_{k} \left(\frac{\partial \Phi}{\partial \eta} \frac{\partial \overline{h}}{\partial g_{k}} - \frac{\partial \Phi}{\partial g_{k}} \frac{\partial \overline{h}}{\partial \eta} \right) \right];$$

$$\Phi = \frac{\partial \Phi}{\partial \eta} = 0, \qquad \overline{h} = \overline{h}_{w} = const. \qquad for \qquad \eta = 0,$$

$$\frac{\partial \Phi}{\partial \eta} \rightarrow 1, \qquad \overline{h} = \overline{h}_{e} = 1 - \kappa \qquad for \qquad \eta \rightarrow \infty,$$

$$(\Phi = \Phi_{0}(\eta), \qquad \overline{h} = \overline{h}_{0}(\eta) \qquad for \qquad \kappa = const., \qquad f_{i} = g_{i} = 0).$$

The characteristic function F_m in this case is:

$$F_m = aB^2 - bf_1 + \frac{2}{B} \left(\sum_{k=0}^{\infty} \theta_k \frac{\partial B}{\partial f_k} + \sum_{k=1}^{\infty} G_k \frac{\partial B}{\partial g_k} \right).$$
 (18)

Since the distribution of outer velocity $u_e(s)$ exists neither in the equation system (17) nor in the corresponding boundary conditions, the system (17) is in that sense universal, i.e. generalized.

The solution of this equation system is practically possible only when there is relatively small number of parameters. Therefore, as with similar flow problems [2], the solution is obtained by so called n - parametric approximation. If it is presumed that all the parameters equal zero, starting from the second one, and if the derivatives per the compressibility parameter and per magnetic parameter are neglected, which means that:

$$\kappa \neq 0, \quad f_1 \neq 0, \quad g_1 \neq 0, \quad f_2 = f_3 = \dots = 0, \quad g_2 = g_3 = \dots = 0, \\
\partial/\partial \kappa = 0, \quad \partial/\partial g_1 = 0$$
(19)

the system (17) becomes much simpler. In the so-called three-parametric twice localized approximation the equation system (17) comes down to:

$$\frac{\partial}{\partial \eta} \left(Q \frac{\partial^{2} \Phi}{\partial \eta^{2}} \right) + \frac{aB^{2} + (2 - b)f_{1}}{2B^{2}} \Phi \frac{\partial^{2} \Phi}{\partial \eta^{2}} + \frac{f_{1}}{B^{2}} \left[\frac{\rho_{e}}{\rho} - \left(\frac{\partial \Phi}{\partial \eta} \right)^{2} \right] - \frac{g_{1}}{B^{2}} \frac{\rho_{e}}{\rho} \left(1 - \frac{\partial \Phi}{\partial \eta} \right) \frac{\partial \Phi}{\partial \eta} =$$

$$= \frac{F_{m}f_{1}}{B^{2}} \left(\frac{\partial \Phi}{\partial \eta} \frac{\partial^{2} \Phi}{\partial \eta \partial f_{1}} - \frac{\partial \Phi}{\partial f_{1}} \frac{\partial^{2} \Phi}{\partial \eta^{2}} \right), \tag{20}$$

$$\frac{\partial}{\partial \eta} \left(\frac{Q}{Pr} \frac{\partial \overline{h}}{\partial \eta} \right) + \frac{aB^{2} + (2 - b)f_{1}}{2B^{2}} \Phi \frac{\partial \overline{h}}{\partial \eta} - \frac{2\kappa f_{1}}{B^{2}} \frac{\rho_{e}}{\rho} \frac{\partial \Phi}{\partial \eta} + 2\kappa Q \left(\frac{\partial^{2} \Phi}{\partial \eta^{2}} \right)^{2} +$$

$$+ \frac{2\kappa g_{1}}{B^{2}} \frac{\rho_{e}}{\rho} \left(1 - \frac{\partial \Phi}{\partial \eta} \right) \left(\frac{\partial \Phi}{\partial \eta} \right)^{2} = \frac{F_{m}f_{1}}{B^{2}} \left(\frac{\partial \Phi}{\partial \eta} \frac{\partial \overline{h}}{\partial f_{1}} - \frac{\partial \Phi}{\partial f_{1}} \frac{\partial \overline{h}}{\partial \eta} \right); \tag{20}$$

$$\begin{split} \Phi &= \frac{\partial \Phi}{\partial \eta} = 0, & \overline{h} &= \overline{h}_w & for & \eta = 0, \\ &\frac{\partial \Phi}{\partial \eta} &\to 1, & \overline{h} &= \overline{h}_e = 1 - \kappa & for & \eta \to \infty, \\ (\Phi &= \Phi_0(\eta), & \overline{h} &= \overline{h}_0(\eta) & for & \kappa = const.; & f_1 = g_1 = 0). \end{split}$$

Under the conditions (19) the expression for the characteristic function F_m is much simplified. In three-parametric unlocalized and twice localized approximation this function is determined by the expressions:

$$F_{m} \approx \frac{aB^{2} - bf_{1} + \frac{4\kappa f_{1}}{B} \frac{\partial B}{\partial \kappa}}{1 - \frac{2}{B} f_{1} \frac{\partial B}{\partial f_{1}} - \frac{2}{B} g_{1} \frac{\partial B}{\partial g_{1}}}, \qquad F_{m} \approx \frac{aB^{2} - bf_{1}}{1 - \frac{2}{B} f_{1} \frac{\partial B}{\partial f_{1}}}; \qquad (21)$$

where the correct expression for the function F_m is determined by the relation (18), that is (7).

Numerical solution of the equation system (20) is performed by the method of finite differences, by the "progonka" procedure, where the order of the equation system was first reduced by the usual shift $u/u_e = \partial \Phi/\partial \eta = \varphi$. For the correct solution of the system, a program has been written in the Fortran program. A similar program developed and applied in [5] was used while writing it.

While solving the obtained equation system (20), for the non-dimensional function Q and for the relation of the densities ρ/ρ_e , the following approximate formulas have been used:

$$Q = Q(\overline{h}) \approx \left(\frac{\overline{h}_w}{\overline{h}}\right)^{1/3}, \quad \frac{\rho_e}{\rho} \approx \frac{\overline{h}}{1 - \kappa},$$
 (22)

where the value of Prandtl's number, Pr = 0.712.

These formulas represent relatively rough approximation in the case of ionized gas flow. For the constants a, b, the usual values in the boundary layer theory have been accepted (a = 0.4408, b = 5.7140).

Out of many obtained numerical results only some of them are shown in this paper, in a form of the corresponding diagrams. Diagrams of non-dimensional velocity (Fig. 1, Fig. 2, Fig. 3), non-dimensional enthalpy (Fig. 4, Fig. 5) and of some characteristics of boundary layer (Fig. 6, Fig. 7, Fig. 8) are given here.

First of all, it should be pointed out that, as with other problems of ionized (or dissociated) gas flow, the obtained solutions are expected, logical and acceptable.

As it is seen in the presented diagrams (Fig. 1, Fig. 2, Fig. 3), non-dimensional velocity, for different values of compressibility parameter, convergates fastly towards zero. The graphics non-dimensional enthalpy (Fig. 4, Fig. 5) are logical, but it is also pointed out that the compressibility parameter has a considerable influence on the distribution of enthalpy in the boundary conditions thus defining its value at the outer boundary.

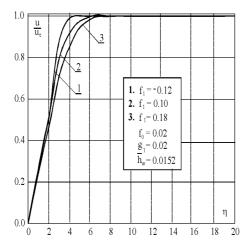


Fig. 1. Graphic of the non-dimensional velocity

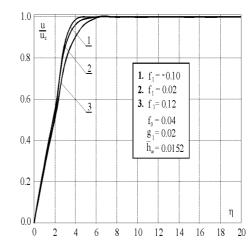


Fig. 2. Graphic of the non-dimensional velocity

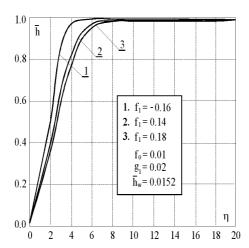


Fig. 3. Graphic of the non-dimensional velocity for different values of the parameter f_0

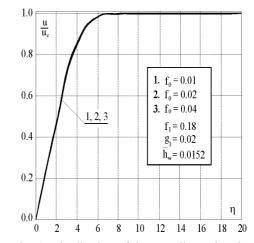


Fig. 4. Distribution of the non-dimensional enthalpy

The behaviour of the boundary layer characteristics (Fig. 6, Fig. 7, Fig. 8) is also expected. However, for some values of the input parameter, an unexpected behaviour of the characteristic F_m (7, 21) in the boundary layer has been noticed even besides making a concrete form of the law of electroconductivity (2).

Further studies should, among other things, give a precise answer to this behaviour of the function mentioned. That's why while solving the obtained equation system (20), as with [5], the correct expression (7) has been used for the characteristic function of the boundary layer F_m .

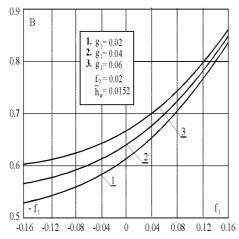


Fig. 5. Distribution of the non-dimensional enthalpy for different values of the parameter f_0

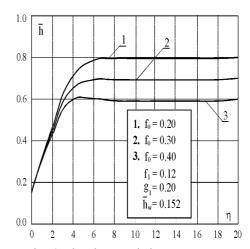


Fig. 6. The characteristic of the boundary layer B

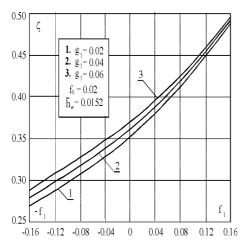


Fig. 7. The characteristic of the boundary layer F_m

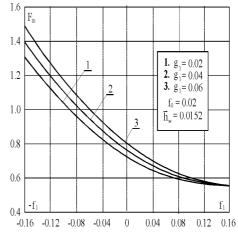


Fig. 8. The non-dimensional function of the friction ζ

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GRANIČNI SLOJ JONIZOVANOG GASA ZA SLUČAJ PROMENLJIVE ELEKTROPROVODNOSTI

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U radu se istražuje strujanje jonizovanog gasa za slučaj konkretnog oblika zakona promene njegove elektroprovodnosti. Odgovarajuće jednačine graničnog sloja su prvo pogodnim transformacijama dovedene na univerzalni oblik. Zatim su jednačine numerički rešene u troparametarskom približenju. Grafički je prikazano ponašanje fizičkih veličina i nekih karakteristika graničnog sloja.